

Body, Emotion, and Calculus

A lesson study on optimization



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Team: Fun with Calculus

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In *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students*, Rochelle Gutiérrez writes, ***“A critical aspect of evidence that mathematics is more rehumanized is that it conjures up feelings of joy.”***

Introduction: Body & Emotion and Optimization

We designed an activity that invites students to explore with their bodies how speed, distance, and time change in relation to one another as they walk across different terrains.

By measuring their own movement using different walking styles, students gather real data and analyze how one quantity affects another before finding the optimal solution using calculus.

Through this practical, hands-on experiment, we aim to deepen students' understanding of the relationships between variables and demonstrate how calculus describes motion in the real world.

Overall Plan

- Interview students to get initial data.
- Pick a topic to design a lesson.
- Executing the first lesson at Long Beach.
- Collect feedback, make necessary adjustment.
- Deliver the revised lesson at Sonoma.
- Analyze the results and draw conclusions.

Listening First: Designing an Optimization Lesson from Student Voices

We began with interviews of focal students. Questions started broadly and moved toward calculus:

- What do you like most about college?
- How would you explain to family or friends what you are learning in calculus?
- One of the examples we work with a lot is moving objects and their velocities. How do you visualize or experience velocity?

Our goal:

Use student language and ways of thinking to shape an optimization lesson centered on body and motion.

What We Learned from Student Interviews

Students often described calculus using phrases like:

- “I imagine...”
- “I picture...”

Most students:

- Could not connect math to their own bodies.
- Did not reference movement, walking, speed, or physical experience.

How This Shaped Our Lesson Design

We intentionally designed an optimization lesson that:

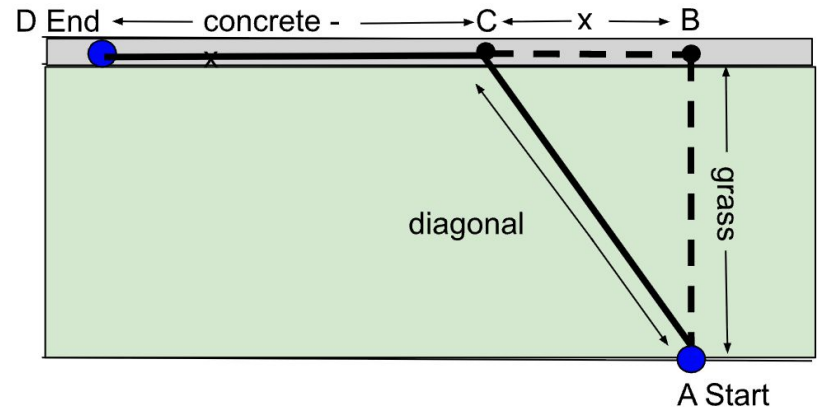
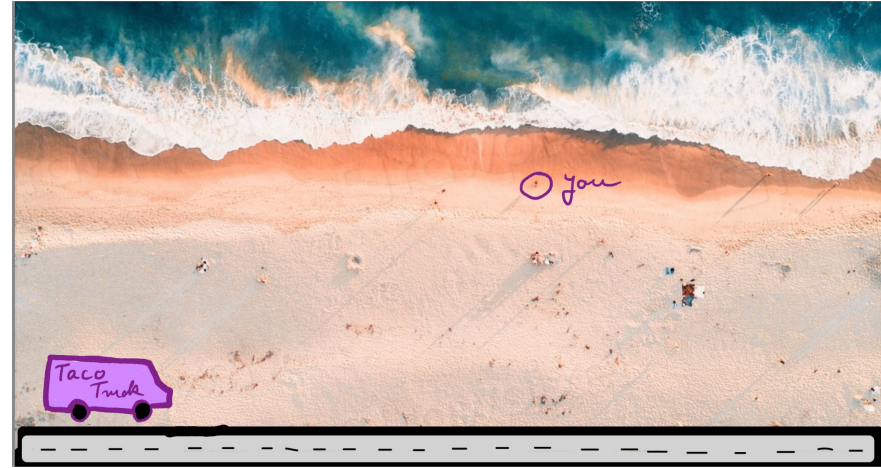
- Begins with physical movement.
- Uses walking, distance, measuring, and surface type (grass vs. concrete).

Math came after experience. Students used their bodies as the first “model”.

Lesson Outline

Phase 1: In-Class Discussion & Preparation

- Discussion:
 - What is the fastest path to the taco truck?
 - How can we simulate the situation outside on campus?
- Introduce the experiment with worksheet where they will fill in collected data
- Provide the equipment such as timers, measuring tape
- Divide students into small groups



Phase 2: Outdoor Data Collection

Class moves outdoors to pre-selected area.

Each group will:

- Measure distances and times to walk three different paths.
- One student walks the distance using two different walking styles (grass/sand and concrete).
- Another student times the walks.
- Record all values carefully.

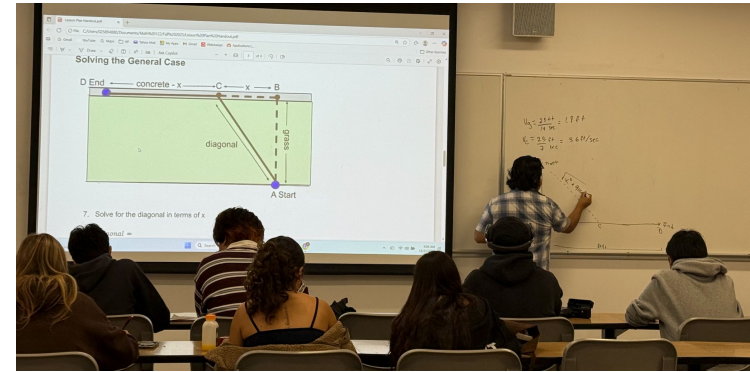
During this phase, the instructor:

- Walks between groups to provide guidance and correct any technical errors.
- Ensure correct measurement.

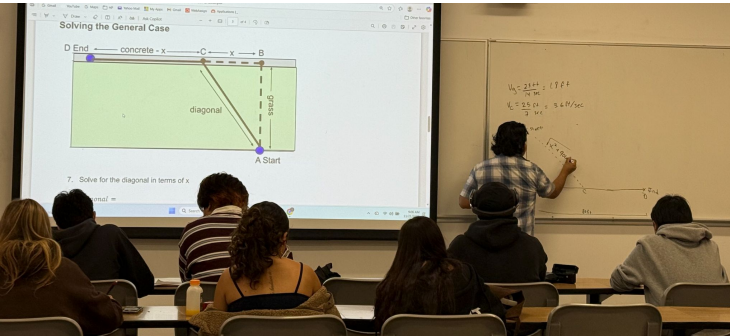


Phase 3: In-Class Analysis and Drawing Connections

- Students fill in the distances and times using collected data from simulation.
- Instructor works through general case on the board:
 - Find time function $T(x)$
 - Find critical values: Solve $T(x)=0$ for x .
 - Compute absolute minimum value of time function.
- Students find the absolute minimum for their own time function.
- Wrap up discussion:
 - Compare the result between theory and real-life data. Observe the difference.
 - Encourage thinking between theory and application.
 - Make connection between theory and experiences using body and motion.



First Lesson: Long Beach in November



Our First Attempt

- Students were energetic and willing to participate in our lesson study
- A couple groups were confused at first on the Instructions
- After the Instructional hurdle all groups were able to begin their experiment
- Data Gathering took longer than expected.
- Gave a lecture on how to use their data to calculate their optimal distance.
- Some were able to finish the calculation
- We were not able to perform the optimal distance calculated.

Revisions Round 1: What we changed

- Time was an issue that we ran into as we did not get to finish our activity.
 - Distribute the material ahead of time instead of letting students retrieve that material on their own
 - Have students measure to the nearest foot or up to 1 decimal place.
- Student felt the instructions were unclear.
 - Due to time constraints instruction were rushed, so we decided to do the pre-discussion ahead of time.

Second Lesson: Sonoma in February

Discuss in your group:
How can we simulate walking on sand?

**Penguin
Waddle!**



Lesson Reflection

- Simulations and measurements went smoothly.
- It was a relatively cold day, which distracted some students.
- We moved to outside “Math Deck” for students to finish recording their data and have the general case mini-lecture.
- Students worked well together to work through their own optimization problems.
- Algebra was still difficult - real data leads to messy equations.
- Students were very invested in finding their solutions
- We still ran out of time, but.....

Next class: Introducing global max/min

$f_{\text{press}} \frac{d}{v} + \text{Consist} \frac{d}{v}$
 $f(x) = \frac{\sqrt{x^2 + 289}}{2} + \frac{30-x}{3}$

$g(x) = \frac{\sqrt{x^2 + 484}}{2} + \frac{29-x}{3}$

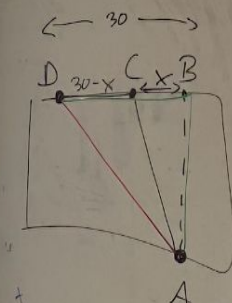
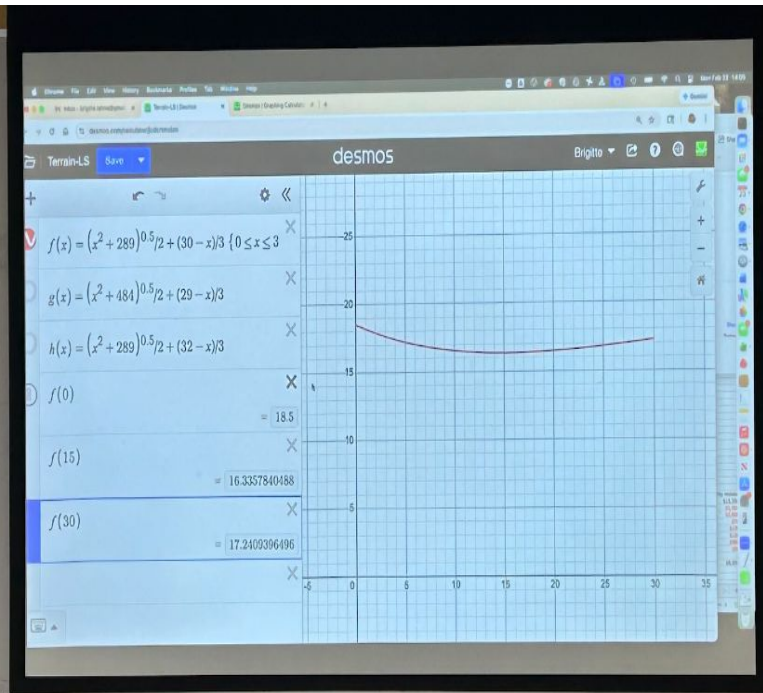
$h(x) = \frac{\sqrt{x^2 + 289}}{2} + \frac{32-x}{3}$

$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2 + 289}} \cdot 2x - \frac{1}{3} = 0$
 $= \frac{x}{\sqrt{x^2 + 289}} - \frac{1}{3} = 0$
 $x = 15 \text{ ft}$

$x \approx 20 \text{ ft}$

$x \approx 15 \text{ ft}$

Critical values



Find the absolute min of $f(x)$ (time) on $0 \leq x \leq 30$:

Critical pt: $x = 15$

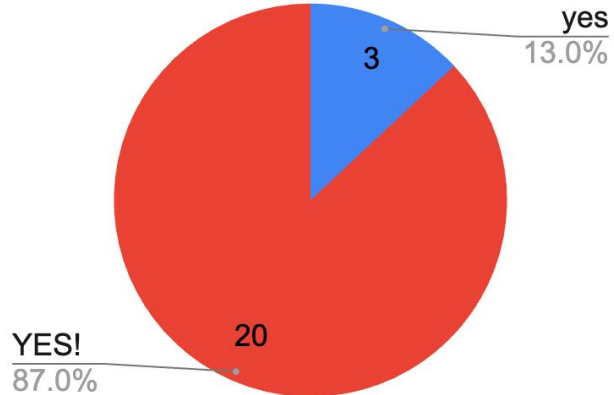
x	$f(x)$
0	18.5 sec (absolute max)
15	16.3 sec (absolute min)
30	17.2 sec

Revisions Round 2: Possible Adaptations

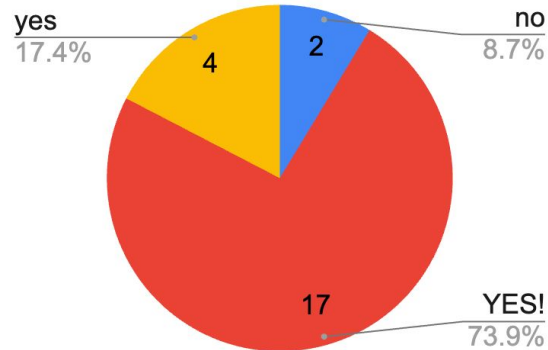
- Real data is messy, so after setting up the time function, let students use technology to find derivatives and critical values.
- Turn the activity into a multi-day project where students collect data outside of class.
- Simplify data collection/recording: We don't really use all the data we asked the students to collect (this also took a lot of time).

Student Feedback: Enjoyment

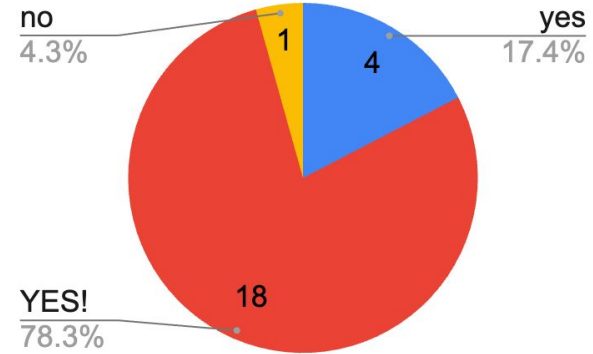
Today's activity was engaging compared to a typical math lesson.



Physically acting out problems helped me feel more connected to the content.

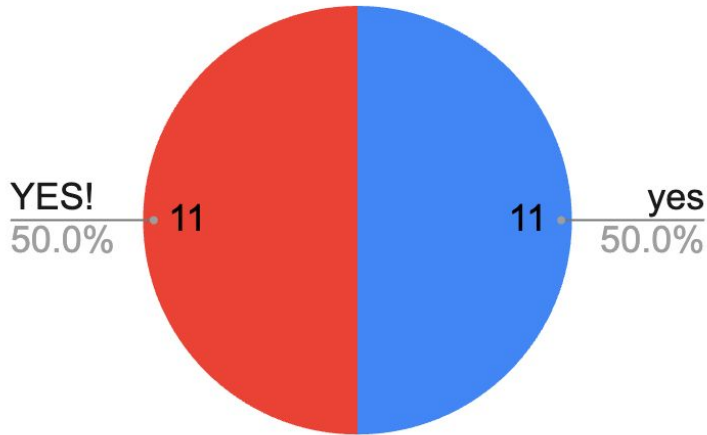


Would you like to do more hands-on math activities like this?

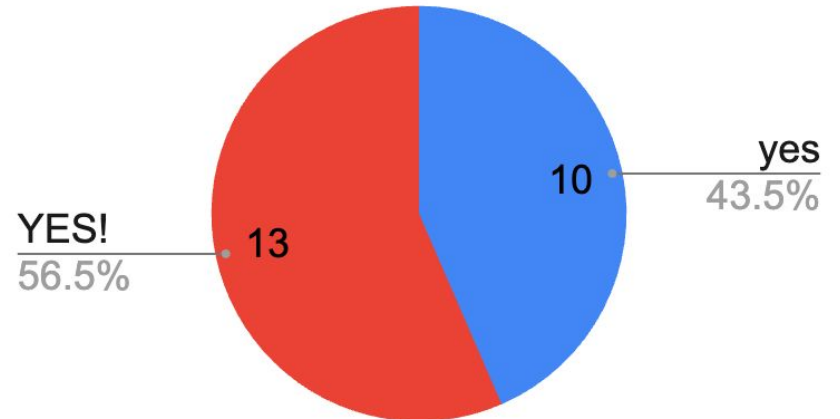


Student Feedback: Clarity

The instructions of the activity were clear.



The goals of the activity were clear.



Student Feedback: If a friend missed class today, how would you explain what you learned to them?

If a friend missed class today, **I would explain to them that we optimized a path based off time consumption on different terrains.** We need to try to get to a certain point, but there is also a terrain that consumes twice as much time as walking on the concrete. You have to try and **optimize a path to take up the least amount of time.**

I would explain it as we used an **outdoor hands-on math experiment to explain optimization.** We discussed which path would be the fastest whether we walked in sand, diagonally, straight, and or a mix of everything. We used meters and time in order to calculate the optimal pathway regarding time.

We went outside, acted out a scenario in which we're walking on sand, we **waddled on the sand, walked normally on the concrete,** and then **computed the distance and velocity.**

I would tell them that we had to find the calculations for the **feet per second from walking on the grass versus walking on the concrete.** We did this by **measuring and timing the distance** and then using d/t to get our answers.

Student Feedback: Free response

I enjoyed today's activity because I didn't [know] that we can use derivative in real life.

It was a really fun activity and we should definitely do more hands-on math, I feel it helped me pay attention better

It was fun, the weather went against the quality of the lesson because the cold temperatures were distracting.

I think that this hands on activity was pretty useful and a fun way to learn the material without just sitting in a lecture.

I think that these interactive activities are quite enjoyable as they make us connect the concepts we learn in class to what we were doing.

I think today's lesson was a great example to show what we're learning class and why what it matters.

I enjoyed being able to have a hands on activity because we were able to see how we could use math in the real world.

The outdoor optimization activity was fun and helped me understand the topic better. Walking different paths and comparing the distances made the idea of optimization much easier to understand. I liked that it was hands-on and showed how different routes can change how far or easy something feels. It was a simple but helpful way to learn the concept.

It was fun understanding optimization in the real-world.

I thought it was interesting and it's a math lesson that I could share with someone else.

Other body & emotion activities we have done or are planning to do

- Padlet posts inspired by the lesson study - I now start more lessons with experience before equations, ask more reflection questions, etc.
- In summary, this helped me shift my teaching toward **Experience** → **Reflection** → **Formalization** - This experience also inspired the content I deliver in my meetings: [Rehumanizing Math](#)

Sanaa Saykali + 30 • 20d **Frozen**

MAT 1150 "So... What Are You Even Learning in That Math Class?"

Imagine your family member, roommate, or friend (who totally zones out when math comes up) asks you: "Wait, what do you actually learn in your Calculus II class?" How would you explain it so they get it - or at least think it sounds cool? 🤪 You can:

- Compare it to something in real life (music, motion, patterns, or puzzles)
- Use a funny analogy or everyday example (add a GIF, a picture from your notes, etc)
- Or describe how what we do in class connects to real-world problems. ✨ Be creative, clear, and maybe a little dramatic - your goal is to make them say, "Ohhh, that actually makes sense!"

Trust me i'm an engineer

I got to learn more about calculus and its subjects such as Taylor and Maclaurin series which will help me apply it to real world issues in my engineering classes such problems that involve patterns and limits.

Responsible Crocodile
3 months ago

I use this gif all the time to let my friends or boyfriend know I'm studying without saying it outright

We calculate lots of things like areas under a curve or infinite series and sequences based on

A car's speed sometimes gets slow and sometimes gets fast, and what I learn is a fast way to calculate how far it has traveled in one hour. Integral will split one hour into a lot of moment, figure out the short distance traveled in each moment, and finally add up all these short distances. I get the total distance.

If I were to explain the topics of power series and functions I would tell them that it is recognizing patterns and applying formulas to certain situations. Just like in real life when you may see a certain situation and apply something you know will work to

0,04880694143167

We find patterns in numbers and test the pattern to see if we kept the pattern going forever, would it become a singular number. It can look pretty cool when graphed too

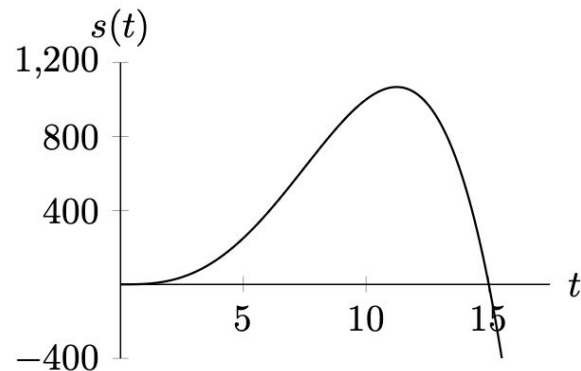
Other body & emotion activities we have done or are planning to do

- Use toy cars (hot wheels) to act out typical calculus problems involving distance, velocity, acceleration.
- Students act out situations given in distance or velocity graphs before answering typical calculus questions.

Example 6. A jogger is running along the side of a straight east-west highway. Her displacement (in meters) from her home is given by

$$s(t) = 3t^3 - 0.2t^4,$$

where t is measured in minutes. Positive values of $s(t)$ indicate that she is east of home, while negative values of $s(t)$ indicate that she is west of home.



Another Class Example for Body/Emotions

Background

- In *Rehumanizing Mathematics for Black, Indigenous, and Latinx Students*, Rochelle Gutiérrez writes, “***A critical aspect of evidence that mathematics is more rehumanized is that it conjures up feelings of joy.***”

Goal

- Create a more supportive and joyful learning environment in the math classroom

Methods

- Begin the class with the deep breathing and participate in a short emotional check-in.
- End the class with the deep breathing and participate in a short emotional check-out.

Survey Results

Forty-four students responded from three classes:

- One Math Majoring Class
- One Upper-Division General Education (GE) Math Class
- One Lower-Division GE Math Class



Continue to Another Class Example for Body/Emotions

Survey Results:

“Those deep breaths before and after class do really help as well!”

#1. Before today’s class, how did you feel about math?

- Excited: 3/44, Calm: 13/44, Neutral: 8/44, A Little Anxious: 14/44, and A Very Anxious: 6/44.

#2. After today’s class, how do you feel about math?

- Excited: 7/44, Calm: 18/44, Neutral: 7/44, A Little Anxious: 8/44, Very Anxious: 4/44

#3. The breathing exercise at the beginning of class helped me feel more focused.

- Strongly Agree: 12/44, Agree: 26/44, and Neutral: 6/44

#4. The check-in/check-out helped me feel heard in class.

- Strongly Agree: 17/44, Agree: 18/44, and Neutral: 9/44

#5. Do these practices reduce your math anxiety?

- Yes: 28/44, A little: 11/44, Not Sure: 4/44, and No: 1/44

Thank you

For more information contact:

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